Close Tue: 14.7(2),15.1
Close Next Thu: 15.2, 15.3 (integrating!)
Close Tue, May 16: 15.4, 15.5 (finish early)
Exam 2, May 16th
Office Hours Today: 1:30-3:00pm (Com B-006)

Entry Tasks: Here are three old exam problems about local max/min, I will do one in class, see how far you can get before we start:

1. Find and classify all critical points for
$$f(x,y) = x^2 + 4y - x^2y + 1$$

2. Find and classify all critical points for
$$f(x,y) = \frac{9}{x} + 3xy - y^2$$

3. Find and classify all critical points for
$$f(x,y) = x^2y - 9y - xy^2 + y^3$$

Global Max/Min

Consider a surface $f(x, y)$ over a particular region R on the xy-plane.

An **absolute/global maximum** over R is the largest z-value over R.

An **absolute/global minimum** over R is the smallest z-value over R.

Key fact (Extreme value theorem)

The absolute max/min must occur at either

1. A critical point, or
2. A boundary point.

Example: Let R be the triangular region in the xy-plane with corners at (0,-1), (0,1), and (2,-1).

Find the absolute max and min over this region R for the surface:

$$f(x, y) = \frac{1}{4}x + \frac{1}{2}y^2 - xy + 1$$

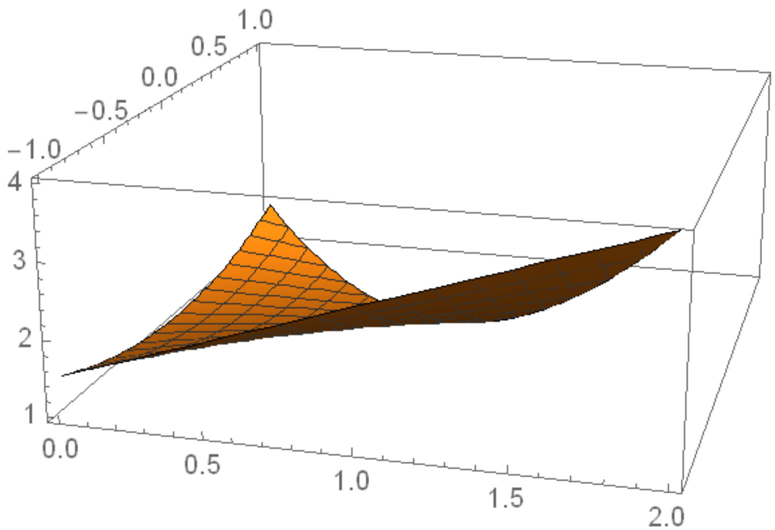
Step 1: Critical points inside region.

Step 2: Boundaries (the triangle has 3).
For each boundary:

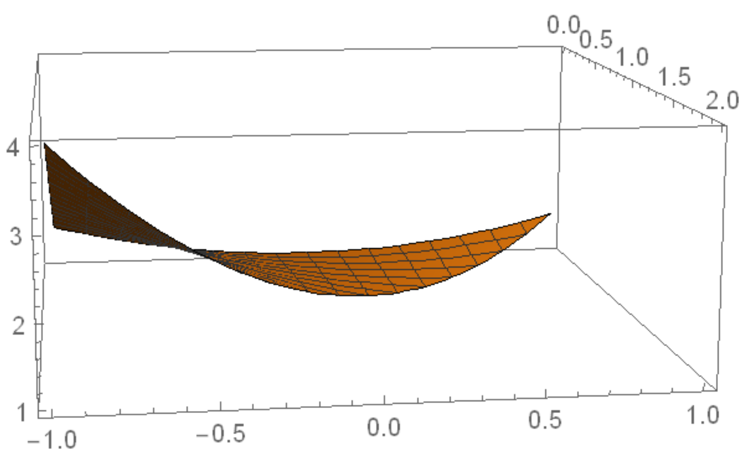
i) Give equation in terms of x and y.
ii) Intersect with surface (substitute).
iii) Find critical numbers and endpoints ("corners") for this one variable function.

Step 3: Evaluate the function at all points you found in steps 1 and 2.

Biggest output = global max
Smallest output = global min



1.0
0.5
0.0
−0.5
−1.0
4
3
2
1
0.0
0.5
1.0
1.5
2.0



0.0
0.5
1.0
1.5
2.0
4
3
2
1
−1.0
−0.5
0.0
0.5
1.0

In applied optimization problems,

(a) Label Everything.
(b) Optimizing what? (objective)
(c) Any given facts? (constraints)
(d) Use the constraints and labels to give a **2 variable function for the thing you are optimizing** (your objective).
(e) Find the critical points **of your objective**.

HW Examples:

1. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to (4,2,0).

Objective: Minimize **distance** from (x,y,z) points on the cone to the point (4,2,0) given that $z^2 = x^2 + y^2$.

2. Find the dimensions of the box with volume 1000 cm^3 that has minimum surface area.

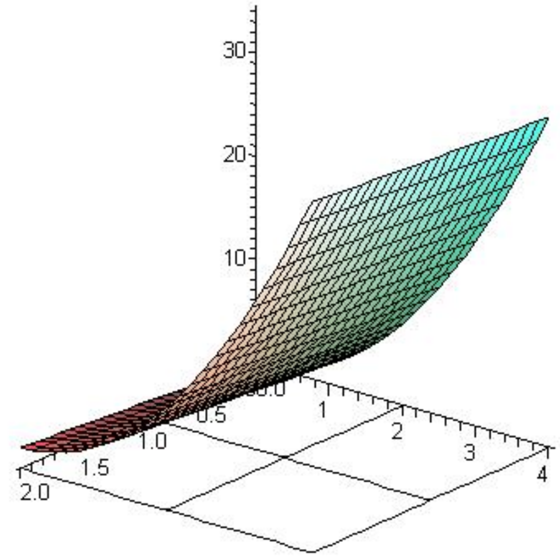
Objective: Minimize **surface area** given that volume is 1000.

3. The bottom of an aquarium is made of slate which cost $2 per square inch. The sides are made of glass and costs $20 per square inch. There is no top. Find the dimensions of the aquarium with volume 100 sq. inches that minimizes cost.

Objective: Minimize **cost** given that volume is 100.

15.1/ 15.2 Double Integrals over Rectangles

Goal: Give a definition for volume “under” a surface and write this volume in terms of integrals.



Example:

Consider the volume under the surface

$$z = f(x, y) = x + 2y^2$$

and above the rectangle

R = [0,2] x [0,4] = {(x,y) : 0 ≤ x ≤ 2, 0 ≤ y ≤ 4 }

Let’s approximate this volume.

(a) Draw the region R in the xy-plane and break it into 4 sub-regions;
m = 2 columns and n = 2 rows.

(b) Approximate using a rectangular box over each region.

In general, we define:

$$\iint_R f(x,y)dA = \lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}, y_{ij})\Delta A$$

= the `signed' volume between *f(x,y)* and the *xy*-plane over *R*.

If *f(x,y)* is above the xy-plane it is positive.

If *f(x,y)* is below the xy-plane it is negative.

General Notes and Observations:

z = *f(x,y)* = height on surface

R = the region on the *xy*-plane

ΔA = area of base = $\Delta x \Delta y = \Delta y \Delta x$

$f(x_{ij}, y_{ij})\Delta A$ = (height)(area of base)

= volume of one approximating box

Units of $\iint_R f(x,y)dA$ are

(units of f(x,y))(units of x)(units of y)

Other quick applications:

$$\iint_R 1dA = \text{Area of R} \quad \text{and}$$

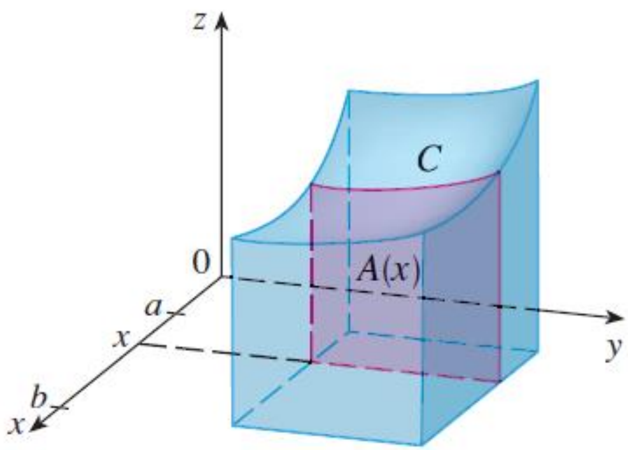
$$\frac{1}{\text{Area of R}} \iint_R f(x,y)dA = \text{Average value of f(x, y) over R}$$

15.2 Using Iterated Integrals to Compute

If you fix x:

The area under this curve is given by

$$\int_c^d f(x,y)dy = \text{"cross sectional area under the surface at this fixed } x \text{ value"}$$



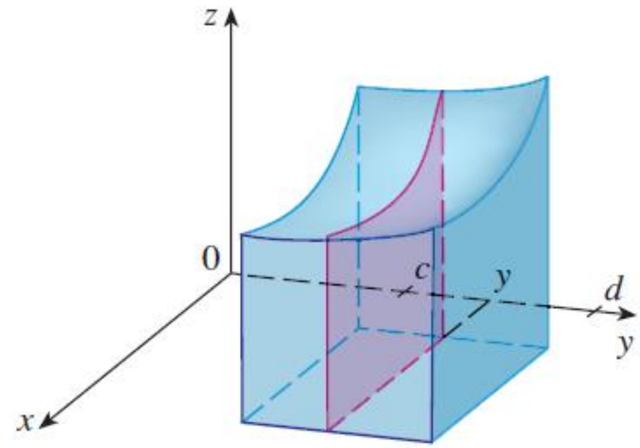
From Math 125,

$$\text{Vol} = \int_a^b \text{Area(x)}dx = \int_a^b \left(\int_c^d f(x,y)dy \right) dx$$

We could also do the other direction.

If you fix y: The area under this curve is given by

$$\int_a^b f(x,y)dx = \text{"cross sectional area under the surface at this fixed } y \text{ value"}$$



$$\text{Vol} = \int_c^d \text{Area}(y)dy = \int_c^d \left(\int_a^b f(x,y)dx \right) dy$$

Examples (like 15.2 HW):

1. Find the volume under $z = x + 2y^2$ and above the rectangular region

$$0 \le x \le 2, \quad 0 \le y \le 4$$

2. $\int_0^3 \int_0^1 2xy\sqrt{x^2 + y^2}\,dx\,dy$

3. Find the double integral of

$$f(x, y) = y\cos(x + y)$$

over the rectangular region

$$0 \leq x \leq \pi, \ \ 0 \leq y \leq \pi/2$$